

Quasispherical gravitational collapse in 5D Einstein-Gauss-Bonnet gravity

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We obtain a general five-dimensional quasispherical collapsing solutions of irrotational dust in Einstein gravity with the Gauss-Bonnet combination of quadratic curvature terms. These solutions are generalization, to Einstein-Gauss-Bonnet gravity, of the five-dimensional quasispherical Szekeres like collapsing solutions in general relativity. It is found that the collapse proceed in the same way as in the analogous spherical collapse, i.e., there exists a regular initial data such that the collapse proceed to form naked singularities violating cosmic censorship conjecture. The effect of Gauss-Bonnet quadratic curvature terms on the formation and locations of the apparent horizon is deduced.

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I. INTRODUCTION

One of the remarkable special classes of the exact solution of the nonvacuum Einstein equations $G_{ab} = \kappa T_{ab}$ available to date was discovered by Szekeres [1]. They are obtained by solving Einstein equations with irrotational dust $T_{ab} = \epsilon u_a u_b$ as the source, for the line element

$$ds^2 = -dt^2 + X^2 dr^2 + Y^2(dx^2 + dy^2), \quad (1)$$

relative to comoving coordinates. Here u_a is a velocity (i.e. unit timelike) vector field and ϵ is the energy density of the system. One can say that Szekeres solutions are obtained when the spherical symmetry orbits in the Lemaitre-Tolman-Bondi (LTB) model [2] are made nonconcentric to destroy the symmetry, but the energy-momentum tensor is still that of dust. Therefore, the Szekeres spacetime is often called as quasispherical, for definiteness we shall name it as quasispherical Szekeres (QSZ) solutions. Being an exact model of the spacetime geometry, the QSZ solutions have primarily been adapted with regards to studies of nonspherical collapse of inhomogeneous dust cloud [1, 3–8] in four dimensions (4D), and in higher dimensions [9, 10], in cosmology [11–16] and also in observational cosmology [17]. They are very important because they admit no Killing vectors [11] and hence they are lacking symmetry.

In any attempt to perturbatively quantize gravity as a field theory, higher-derivative interactions must be included in the action. Such terms also arise in the effective low-energy action of string theories. Among the higher curvature gravities, the most extensively studied theory is the so-called Einstein-Gauss-Bonnet (EGB) gravity [18–29]. The EGB gravity is a special case of Lovelocks' theory of gravitation, whose Lagrangian contains just the first three terms. Gauss-Bonnet gravity provides one of the most promising frameworks to study curvature corrections to the Einstein action in supersymmetric string

theories, while avoiding ghosts and keeping second order field equations.

In Einstein-Gauss-Bonnet gravity less number of exact solutions have been known so far. Static and spherically symmetric black hole solutions with or without a cosmological constant, as well as a topological black hole in an anti-de-Sitter spacetime were obtained [18–22]. The effects of Gauss-Bonnet terms on the Vaidya solutions have been investigated in [23–27], and on the LTB solutions in [28, 29]. Here, we consider the five-dimensional (5D) action with the Gauss-Bonnet terms for gravity and give an *exact model* of the quasispherical gravitational collapse. Using our new solution, we investigate the nature of singularities of such a spacetime in terms of its being hidden within a black hole, or whether it would be visible to outside observers and compare it with analogous relativistic case. Thus, the aim of this paper is to extend the previous studies on the quasispherical gravitational collapse of inhomogeneous dust, including the second order perturbative effects of quantum gravity solutions: a Gauss-Bonnet generalization of the QSZ solutions in 5D spacetime, namely, 5D-QSZ-EGB.

There are several issues that motivate our analysis: how does the Gauss-Bonnet term affect the final fate of collapse? What is the horizon structure in the presence of the second order perturbative effects of quantum gravity? Whether such solutions lead to naked singularities? Do they get covered due to departure from spherical symmetry? Does the nature of the singularity changes in a more fundamental theory preserving cosmic censorship [41]?

II. SZEKERES SOLUTIONS IN 5D EINSTEIN GAUSS-BONNET GRAVITY

In this section, we derive the relevant equations for Szekeres model for irrotational dust to the 5D Gauss-Bonnet extended Einstein equation – 5D-QSZ-EGB solutions. We begin with the following 5D action:

$$S = \int d^5x \sqrt{-g} \left[\frac{1}{2\kappa_5^2} (R + \alpha L_{GB}) \right] + S_{matter}, \quad (2)$$

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where R is 5D the Ricci scalar, and $\kappa_5 \equiv \sqrt{8\pi G_5}$ is 5D gravitational constant which is set to unity. The Gauss-Bonnet Lagrangian is of the form

$$L_{GB} = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}, \quad (3)$$

where α is the coupling constant of the Gauss-Bonnet terms. This type of action is derived in the low-energy limit of heterotic superstring theory [30]. In that case, α is regarded as the inverse string tension and positive definite, and we consider only the case with $\alpha \geq 0$ in this paper.

The action (2) leads to following set of field equations:

$$\mathcal{G}_{ab} \equiv G_{ab} + \alpha H_{ab} = T_{ab}, \quad (4)$$

where

$$G_{ab} = R_{ab} - \frac{1}{2}g_{ab}R, \quad (5)$$

is the Einstein tensor and

$$H_{ab} = 2[RR_{ab} - 2R_{a\alpha}R_b^\alpha - 2R^{\alpha\beta}R_{a\alpha b\beta} + R_a^{\alpha\beta\gamma}R_{b\alpha\beta\gamma}] - \frac{1}{2}g_{ab}L_{GB}, \quad (6)$$

is the Lanczos tensor.

To find QSZ spacetime, Szekers [1] imposed no *a priori* symmetry assumption, but instead postulated the metric in a special form which, for our 5D case, can be written as:

$$ds^2 = -dt^2 + X^2 dr^2 + Y^2(dx^2 + dy^2 + dz^2). \quad (7)$$

The geodesic fluid flow vector is $\partial/\partial t$ and the coordinates (x, y, z) are comoving spatial coordinate constant along each world-lines so that $u^a = \delta_t^a$. X and Y are functions of (t, x, y, z) to be determined from the Einstein equations. The energy-momentum tensor for dust is

$$T_{ab} = \epsilon(t, x, y, z)\delta_a^t\delta_b^t. \quad (8)$$

It is seen that there exist solutions according as $Y' = 0$ or as $Y' \neq 0$ (throughout this paper $' \equiv \partial/\partial t$ and $\partial/\partial r$). In 4D, the family of the solution corresponding to $Y' = 0$ is a coincidental generalization of the Friedman and Kantowski-Sachs models. Bolejko *et al.* [16] pointed out, in 4D, that the case $Y' = 0$ found no useful application in astrophysical cosmology. Hence, we shall confine our discussion to the case $Y' \neq 0$. After the Einstein equations are solved, it is required that there must exist two functions $R(t, r)$ and $\nu(r, x, y, z)$ such that

$$Y = \frac{R(t, r)}{P(r, x, y, z)}, \quad (9)$$

$$X = \frac{PY'}{W(r)}. \quad (10)$$

Here, $W = W(r)$ is an arbitrary function of r with restriction that $W(r) > 0$. The solution for P reads

$$P = A(r)(x^2 + y^2 + z^2) + B_x(r)^2 x + B_y(r)^2 y + B_z(r)^2 z + C(r), \quad (11)$$

with the free functions $A(r)$, $B_x(r)$, $B_y(r)$, $B_z(r)$ and $C(r)$ satisfying an algebraic equation

$$AC - B_x^2 + B_y^2 + B_z^2 = \frac{\epsilon}{4}, \quad \epsilon = 0, \pm 1. \quad (12)$$

The factor ϵ determines whether the 2-surface is spherical ($\epsilon = +1$), pseudospherical [hyperbolic] ($\epsilon = -1$) or planar ($\epsilon = 0$) [14]. Upon introducing transformations

$$\begin{aligned} x &= \sin \psi \sin \phi \cot \frac{\theta}{2}, \\ y &= \cos \psi \sin \phi \cot \frac{\theta}{2}, \\ z &= \cos \phi \cot \frac{\theta}{2}, \end{aligned} \quad (13)$$

the 3-metric yields the more familiar spherical form

$$R^2(t, r)(d\theta^2 + \sin^2 \theta(d\phi^2 + \sin^2 \phi d\psi^2)). \quad (14)$$

As can be seen, if $t = \text{const}$, and $r = \text{const}$, the above metric becomes the metric of the three-dimensional sphere. Hence, every $t = \text{const}$, and $r = \text{const}$ slice of the Szekeres spacetime is a sphere of radius R . Thus the term quasi-spherical adapted. However, the spheres under consideration are not concentric. Thus, the QSZ solutions considered here are a generalization of the 5D-LTB solutions in which sphere of constant mass are made nonconcentric. The $A(r)$, $B_x(r)$, $B_y(r)$, $B_z(r)$ and $C(r)$ determine how the center of a 3-sphere changes its position in a $t = \text{constant}$ space when the radius of the sphere is changed. When the spheres are concentric, the metric (7) becomes the line element of the 5D-LTB model [31–34] which arises when we set $A = C = 1/2$, $B_x = B_y = B_z = 0$, with no loss of generality.

The acceleration equation is given by the $\mathcal{G}_r^r = 0$ component of the field equations

$$\frac{\ddot{R}}{R} \left[1 - 4\alpha \frac{(W^2 - 1)}{R^2} + 4\alpha \frac{\dot{R}^2}{R^2} \right] - \frac{(W^2 - 1)}{R^2} + \frac{\dot{R}^2}{R^2} = 0. \quad (15)$$

Thus, it is necessary that R satisfies the *Friedmann*-like equation

$$\dot{R}^2 \left[1 - 4\alpha \frac{W^2 - 1}{R^2} \right] = (W^2 - 1) + \frac{F}{R^2} - 2\alpha \frac{\dot{R}^4}{R^2}. \quad (16)$$

Here, $F = F(r)$ is an arbitrary function of r and is referred to as mass function. In a Newtonian limit $F(r)$ is equal to the mass inside the shell of radial coordinate r , assuming that the mass function has no angular dependence. The function F must be positive, because $F < 0$ implies the existence of negative mass. Equation (16) is

the master equation of the system which governs the dynamical properties of the system, which is the same as one obtains in a spherically symmetric collapse of dust in EGB [28]. Hence, we can refer to the 5D-QSZ-EGB discussed here as quasispherical collapsing space-times.

It is now straight forward to calculate the energy density ϵ , which we obtain by substituting the solutions for X and Y . The mass density is

$$\epsilon(t, x, y, z) = \frac{3}{2} \frac{PF' - 4FP'}{P^3 XY^3} = \frac{3}{2} \frac{PF' - 4FP'}{R^3 (PR' - RP')}. \quad (17)$$

This solution has in general no symmetry. For the spher-

ical symmetry these solution corresponds to more convenient 5D Tolman-Bondi models [32, 33].

In many instance, it may be preferential to use in our work not density ϵ , but mass M . M is mass within sphere ($r=\text{constant}$) at given time t , which we compute as follows

$$M(t, r) = \int_0^r dr \int \int \int dx dy dz XY^3 \epsilon. \quad (18)$$

Upon using the density Eq.(17) in (18), we get

$$M(t, r) = \int_0^r dr \frac{3}{2} \int \int \int dx dy dz \frac{PF' - 4FP'}{WP^3} \quad (19)$$

$$= \int_0^r \frac{3}{2} \left[\frac{F'}{W} \int \int \int \frac{dx dy dz}{P^2} + \frac{2F}{W} \frac{d}{dr} \int \int \int \frac{dx dy dz}{P^2} \right] \quad (20)$$

Now $dx dy dz / P^2$ is a metric on the unit 3-sphere, so

$$\int \int \int \frac{dx dy dz}{P^2} = 2\pi^2 \quad (21)$$

so that

$$\frac{dM}{dr} = 2\pi^2 XY^3 \epsilon \quad (22)$$

and

$$M(t, r) = 3\pi^2 \int_0^r \frac{F'}{W} dr \quad (23)$$

This is a generalization of a formula obtained for mass function in 4D QSZ [1]. In addition, $\epsilon > 0$ implies $dM(r)/dr > 0$ or $F'(r)W(r) > 0$.

It is easy to see that as $\alpha \rightarrow 0$ the master solution (16) of the system reduces to the corresponding 5D-QSZ solution in [9, 10]

$$\dot{R}^2 = W^2 - 1 + \frac{F}{R^2}. \quad (24)$$

Rotation and acceleration of the dust source are zero, the expansion is

$$\Theta = -\frac{\dot{Y}'}{Y'} - 3\frac{\dot{R}}{R}, \quad (25)$$

and the shear is

$$\Sigma^2 = \frac{1}{9} \left[7\frac{\dot{Y}'}{Y'^2} - 12\frac{\dot{Y}'\dot{R}}{R'R} + 9\frac{\dot{R}^2}{R^2} \right]. \quad (26)$$

The 5D-LTB limits of these scalars can be obtained by replacing Y by R in the above expressions.

We assume, as in QSZ models [1], the following regularity conditions:- 1. The metric is C^1 . 2. There are no shell crossing singularities ($Y' = PR' - RP' > 0$). 3. The metric is locally Euclidean, i.e., we must have $W(0) = 1$.

The dynamical properties of the model discussed above is governed by Eq. (16), which is the same as one gets in the corresponding spherical symmetric collapsing solutions [28]. To get the 5D homogeneous dust limit, we put $W^2(r) = 1 + k_0 r^2$, $F(r) = Kr^3$, where $k_0 = 0, \pm 1$ and K is constant. Then one gets three type of solutions - parabolic, elliptic or hyperbolic according as $W^2(r) = 1$, $W^2(r) < 1$ or $W^2(r) > 1$.

In particular, the 5D Friedmann-Robertson-Walker metric follows when $R(t, r) = S(r)f(t)$ and $W(r) = 1 - k_0 S^2(r)$, where k_0 is the curvature index of 5D Friedmann-Robertson-Walker models and $B_x = B_y = B_z = 0$, $C = 4A = 1$. Then the metric (7) obtains usual form

$$ds^2 = -dt^2 + \frac{f^2(t)}{1 - k_0 S^2} dS^2 + S^2 f^2(t) d\Omega_3^2. \quad (27)$$

A. Solutions for the zero-energy case, $W(r) = 1$

It may be noted that in the general relativistic case ($\alpha \rightarrow 0$), Eq. (16) has three types of solutions and $W(r) > 1$, $W(r) = 1$ or $W(r) < 1$ determines the type of evolution. From Eq. (16), we obtain

$$\dot{R}^2 = (W^2 - 1) - \frac{R^2}{4\alpha} \left(1 \mp \sqrt{1 + \frac{16\alpha^2}{R^4} (W^2 - 1)^2 + \frac{8\alpha F(r)}{R^4}} \right). \quad (28)$$

There are two families of solutions that correspond to the sign in front of the square root in Eq. (28). We call the family that has the minus (plus) sign the minus (plus) branch solution. In the general relativistic limit $\alpha \rightarrow 0$, we recover the 5D-QSZ solution in Einstein gravity [9, 10]. Maeda [29] has analyzed LTB models near center ($r \sim 0$) in EGB and pointed out the occurrence of major changes in the final fate of collapse (see also, [28]). Here we present the 5D-QSZ-EGB exact solution in close form, which facilitates us to analyze the final fate of gravitational collapse. The condition $W(r) = 1$, is the marginally bound condition, meaning the collapsing shell is at rest at spatial infinity ($R = \infty$) with zero energy in the infinite past. In the present discussion, we are concerned with gravitational collapse, which requires $\dot{R}(t, r) < 0$, i.e., we assume that all portion of the dust cloud are momentarily collapsing. Eq. (28) can be integrated to

$$t_\zeta(r) - t = \frac{\sqrt{\alpha}}{2\sqrt{2}} \tan^{-1} \left[\frac{3R^2 - \sqrt{R^4 + 8\alpha F}}{2\sqrt{2}R[\sqrt{R^4 + 8\alpha F} - R^2]^{1/2}} \right] + \sqrt{\frac{\alpha R^2}{\sqrt{R^4 + 8\alpha F} - R^2}}, \quad (29)$$

where $t_\zeta(r)$ is an arbitrary function of integration. Here we note that the solutions $R(t, r)$ are same as the 5D-LTB-EGB models [28] and are not affected by the dependence of the Szekeres model on the (x, y, z) coordinates. As it is possible to make an arbitrary relabeling of spherical dust shells by $r \rightarrow g(r)$, without loss of generality, we fix the labeling by requiring that, on the hypersurface $t = 0$, r coincide with the area radius

$$R(0, r) = r. \quad (30)$$

This corresponds to the following choice of $t_\zeta(r)$:

$$t_\zeta(r) = \frac{\sqrt{\alpha}}{2\sqrt{2}} \tan^{-1} \left[\frac{3 - \sqrt{1 + 8\alpha \tilde{F}}}{2\sqrt{2}[\sqrt{1 + 8\alpha \tilde{F}} - 1]^{1/2}} \right] + \sqrt{\frac{\alpha}{\sqrt{1 + 8\alpha \tilde{F}} - 1}}, \quad (31)$$

where $\tilde{F} = F/r^4$. Now, we discuss the nature and occurrence of curvature singularities in the 5D-QSZ-EGB models. It follows from Eq. (17) that the curvature singularities occurs in 5D-QSZ-EGB solutions when

$$R = 0 \text{ or } Y' = PR' - RP' = 0. \quad (32)$$

Szekeres [1] analyzed the singularities of the first kind and second kinds according to the above two conditions. As is the case of the LTB, in 4D, $Y' = 0$ corresponds to a shell crossing, however it is qualitatively different from that which occurs in LTB models (see [16] for details). In this paper we shall restrict ourselves to the singularities of the first kind which we call the central singularity. Let

us assume at $t = t_c(r)$, we have $R(t, r) = 0$, which is the time when the matter shell $r = \text{constant}$ hits the physical singularity, i.e., $t_c(r)$ is defined by $R(t_c(r), r) = 0$.

The central singularity curve can be obtained using Eq. (29) as

$$t_c(r) = t_\zeta(r) + \frac{\pi\sqrt{\alpha}}{4\sqrt{2}}, \quad (33)$$

which represents the proper time for the complete collapse of a shell with coordinate r . Interestingly, positive α delays the formation of singularity. In the limit of vanishing α we recover the crunch time for relativistic 5D-QSZ-EGB. The eight arbitrary functions $A(r)$, $B_x(r)$, $B_y(r)$, $B_z(r)$, $C(r)$, $F(r)$, $W(r) > 0$ and $t_c(r)$ completely specify the dynamics of collapsing shells. However, only seven are independent because relation (12) is between them. With the choice of r , one can fix one more function and thus the total degree of freedom is six. The corresponding 5D-LTB-EGB models have only three free functions: $F(r)$, $W(r)$ and $t_c(r)$. Hence the 5D-QSZ-EGB is functionally more generic.

In order to study the collapse of a finite spherical body in EGB, we have to match the 5D-QSZ-EGB solution along the timelike surface at some $r = r_c > 0$ to the 5D-EGB Schwarzschild exterior discovered by Boulware and Deser [18], and Wheeler [20]. The analysis is similar to the case of matching 5D-LTB-EGB to the 5D-EGB Schwarzschild exterior [29] and, will not be discussed here. Bonnor [11] was the first to prove that Szekeres spacetime can be matched to Schwarzschild vacuum spacetime.

III. APPARENT HORIZON AND TRAPPED SURFACE

The apparent horizon (AH) is the outermost marginally trapped surface for the outgoing photons. The AH can be either null or spacelike, that is, it can "move" causally or acausally. The main advantage of working with the apparent horizon is that it is local in time and can be located at a given spacelike hypersurface. The event horizon instead is nonlocal. Moreover, the event horizon is an outer covering surface of apparent horizon and they coincide in case of static or stationary spacetime. Trapped surface is defined as a compact spacelike 2-surface both of whose future pointing null geodesics families are converging. Physically, it captures the notion of trapping by implying that if 2-surface $S_{(r,t)}$ ($t, r = \text{constant}$) is a trapped surface then its entire future development lies behind a horizon. To obtain criterion for existence of such $S_{(r,t)}$, let K^a be the tangent vector to the null geodesics. It follows that along null geodesics, we have

$$K_\mu K^\mu = 0, \quad K^\mu_{;\nu} K^\nu = 0, \quad (34)$$

and $K^x = K^y = K^z = 0$. For the metric (7) the above defined null congruence satisfies the following condition:

$$(K^t)^2 - X^2(K^r)^2 = 0, \quad (35)$$

on $S_{(r,t)}$. A choice of affine parameter

$$K^t = X, \quad K^r = \varepsilon, \quad (36)$$

with $\varepsilon = \pm 1$, clearly satisfies the condition (35). The positive (negative) sign of invariant $K^\mu_{;\mu}$ is determines the divergence (convergence) of the null geodesic. Also \pm corresponds to the outward and inward geodesic respectively. Now,

$$K^\mu_{;\mu} = K^t_{;t} + K^r_{;r} + \varepsilon \left(\frac{X'}{X} + 3 \frac{Y'}{Y} \right) + X \left(\frac{\dot{X}}{X} + 3 \frac{\dot{Y}}{Y} \right). \quad (37)$$

Upon taking the time derivative of the null condition $K^\mu K_\mu = 0$, we get

$$K^t_{;t} - \varepsilon X K^r_{;t} - \dot{X} = 0. \quad (38)$$

In the second condition $K^\mu_{;\nu} K^\nu = 0$, consider the $\mu = 1$ component;

$$K^r_{;r} + \varepsilon X K^r_{;t} + \varepsilon \frac{X'}{X} + 2\dot{X} = 0. \quad (39)$$

Eliminating $K^r_{;t}$ from the previous two equations we can rewrite Eq. (37) as

$$K^\mu_{;\mu} = 3\varepsilon \frac{Y'}{Y} + 3X \frac{\dot{Y}}{Y}. \quad (40)$$

Thus the apparent horizon must satisfy

$$\varepsilon Y' + X \dot{Y} = 0. \quad (41)$$

In absence of shell crossings ($Y' \neq 0$), and using Eqs. (9) and (10), we have

$$\dot{R}^2(t_{AH}(r), r) = -W(r). \quad (42)$$

Considering Eq. (16), the apparent horizon condition (42) becomes

$$R(t_{AH}(r), r) = \sqrt{F(r) - 2\alpha}. \quad (43)$$

One can also employ the usual condition for the existence of the apparent horizon

$$g^{ab} Y_{,a} Y_{,b} = 0. \quad (44)$$

Considering Eqs. (9),(10) and (16), the apparent horizon condition (44) again gives the same result (43). In the relativistic limit, $\alpha \rightarrow 0$, $R_{AH} \rightarrow \sqrt{F(r)}$ [32]. Further, Eq. (43) has a mathematical similarity for the analogous situation in null fluid collapse where the expression for apparent horizon is $r_{AH} = \sqrt{m(v) - 2\alpha}$ [27].

IV. END STATE OF COLLAPSE

In this section, we analyze end state of the collapse of 5D-QSZ-EGB dust collapse in terms of the given regular initial density and velocity profile. We denote $\rho(r)$ as the initial density of the dust cloud at $t = 0$ for a fixed radial direction ($x = y = z = \text{constant}$). It is assumed that ρ , P and F to be expandable around $r = 0$ on the initial hypersurface. Further the function P is restricted by Eqs. (11) and (12). Henceforth, in this section, we adopt here a method similar to [3, 5] which we modify here to accommodate the higher dimension spacetime. We take

$$\rho = \sum_{n=0}^{\infty} \rho_n r^n, \quad (45)$$

$$P = \sum_{n=0}^{\infty} P_n r^n, \quad (46)$$

where $P_0 \neq 0$ and the regularity condition implies that $P_1 = 0$ [5], and

$$F = \sum_{n=0}^{\infty} F_n r^{n+4}. \quad (47)$$

Therefore,

$$P' = \sum_{n=2}^{\infty} n P_n r^{n-1}, \quad (48)$$

$$F' = \sum_{n=0}^{\infty} (n+4) F_n r^{n+3}. \quad (49)$$

Substituting P' and F' in the ρ equation one gets,

$$\begin{aligned} F_0 &= \frac{\rho_0}{6}, & F_1 &= \frac{2\rho_1}{15}, \\ F_2 &= \frac{\rho_2}{9}, & F_3 &= \frac{2\rho_3}{21} - \frac{4P_2\rho_1}{105P_0}, \\ F_4 &= \frac{\rho_4}{12} - \frac{1}{P_0} \left(\frac{P_2\rho_2}{18} - \frac{P_3\rho_1}{20} \right), \\ F_5 &= \frac{2\rho_5}{27} - \frac{21}{9P_0}, \\ &\times \left(\frac{4P_4\rho_1}{15} + \frac{P_3\rho_2}{3} + \frac{2P_2\rho_3}{7} + \frac{2P_2^2\rho_1}{105} \right). \end{aligned} \quad (50)$$

This implies that ρ_0, ρ_1, ρ_3 can not have angular dependence whereas ρ_n (for $n \geq 3$) have angular dependence provided $\rho_{n-2} \neq 0$. The results obtained here are consistent with the earlier work [3, 5], but with some changes in the coefficients due to 5D spacetime. Since earlier analysis were done in QSZ spacetime, as a result, one may conclude that the 5D-QSZ-EGB spacetime has same local nakedness behavior as the QSZ spacetime. To conserve

space, we shall avoid replication of all other analysis being similar to QSZ spacetime [3, 5]. To further analyze the horizon curve, we combine Eqs. (29) and (33) giving

$$t_c(r) - t = \frac{\pi\sqrt{\alpha}}{4\sqrt{2}} + \sqrt{\frac{\alpha R^2}{\sqrt{R^4 + 8\alpha F} - R^2}} + \frac{\sqrt{\alpha}}{2\sqrt{2}} \tan^{-1} \left[\frac{3R^2 - \sqrt{R^4 + 8\alpha F}}{2\sqrt{2}R[\sqrt{R^4 + 8\alpha F} - R^2]^{1/2}} \right]. \quad (51)$$

The apparent horizon in the interior of the dust ball lies at $R(t_{AH}(r), r) = \sqrt{F(r) - 2\alpha}$. The corresponding time $t_{AH}(r)$ is given by

$$t_c(r) - t_{AH}(r) = \frac{\pi\sqrt{\alpha}}{4\sqrt{2}} + \frac{\sqrt{\alpha}}{2\sqrt{2}} \tan^{-1} \left[\frac{F - 4\alpha}{2\sqrt{2\alpha(F - 2\alpha)}} \right] + \frac{1}{2}\sqrt{F - 2\alpha}. \quad (52)$$

As mentioned above, at $t = t_c(r)$, we have $R(t, r) = 0$, which is the time when the matter shell $r = \text{constant}$ hits the physical singularity. The singularity is at least locally naked if $t_{AH} > t_c$, and if $t_{AH} > t_c$, it is a black hole, and in case of the equality one has to compare the slopes. We consider the following three cases:

a. Case I Homogeneous case, i.e., the density profile is homogenous and has no angular dependence, thus

$$F = F_0 r^4 \quad (53)$$

Using Eq. (50), it can be deduced that $t_c < t_{AH}$ (see also Fig.1), and hence singularity is naked.

b. Case II Next, we assume that only $F_1 \neq 0$ implies that the density profile is inhomogeneous and has no angular dependence

$$F = F_0 r^4 + F_1 r^5 \quad (54)$$

It is seen that $t_c < t_{AH}$ which leads to formation of a naked singularity.

c. Case III Finally, let $F_3 \neq 0$ which means the density profile is inhomogeneous and has angular dependence

$$F = F_0 r^4 + F_3 r^7 \quad (55)$$

In this case also singularity is naked because $t_c < t_{AH}$ (see also Fig.1).

Clearly, for a positive α , the central shell does not get trapped, and the untrapped region around the center increases with increasing α , for both homogeneous and inhomogeneous models. The center ($r = 0$) remains untrapped, since for nonzero values of the Gauss-Bonnet parameter $\alpha > 0$, above Eq. (43) admits no solution. Interestingly, the theory demands α to be a positive number, which forbids the apparent horizon from reaching the center thereby making the singularity massive and eternally visible, which is forbidden in corresponding general relativistic scenario.

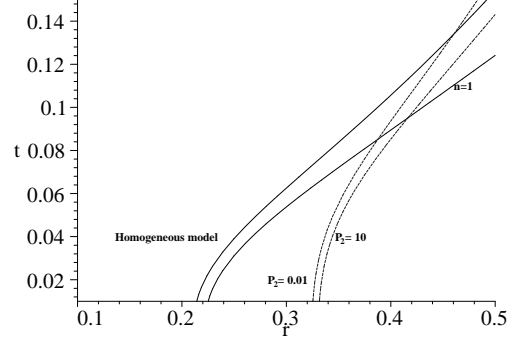


FIG. 1: A two-dimensional picture of the 5D-QSZ-EGB collapse showing $t = t_c(r) - t_{AH}(r)$ versus r for the suitable values of parameters. The continuous curves used when density has no angular dependence and when density has angular dependence dotted curves are used.

V. EFFECT OF THE GAUSS-BONNET TERM

It is seen here that the Gauss-Bonnet term modifies the time of formation of singularity, and the time lag between singularity formation and the apparent horizon formation, in contrast to the 5D dust models. Indeed, the time for the occurrence of the central shell focusing singularity for the collapse is increased in comparison to the 5D-QSZ case. The introduction of a Gauss-Bonnet term slows down the collapse process [28], which can be explained as follows. The contribution from the Gauss-Bonnet term leads, when we think in terms of a Newtonian potential, to a repulsive term. The Eq. (16), for the marginally bound case, becomes

$$\dot{R}^2 = \frac{F}{R^2} - 2\alpha \frac{\dot{R}^4}{R^2}. \quad (56)$$

In the $\alpha \rightarrow 0$ limit we recover the 5D-QSZ solution, and in this limit, Eqs. (16) and (56) lead to the following expression for acceleration:

$$\ddot{R}_{5D-QSZ} = -\frac{F}{R^3}. \quad (57)$$

In the 5D-QSZ-EGB spacetime the acceleration expression (16) can now be expressed as

$$\ddot{R}_{5D-QSZ-EGB} = -\frac{R}{4\alpha} + \frac{R}{4\alpha} \left[1 + 8\alpha \frac{F}{R^4} \right]^{-1/2}. \quad (58)$$

For a comparison with Newton like force equation obtained in the 5D-QSZ case $\Phi(R) = -F/R^3$, clearly a positive value of α diminishes the acceleration in the collapse process. As we move out to larger shells and for large values of α the detailed dynamics should depend on higher order terms in the expansion.

The other reason may be, for slow down of the collapse process in 5S-QSZ-EGB, there is relatively less mass energy [see Eq. (60)] collapsing in the 5D-QSZ-EGB spacetime as compared to the 5D-QSZ case. This can be seen

from the mass function $m(t, r)$, which is given by

$$m(t, r) = R^2 (1 - g^{ab} R_{,a} R_{,b}). \quad (59)$$

Using Eqs. (9), (10) and (16) into Eq. (59) we get

$$m(t, r) = F(r) - 2\alpha \dot{R}^4. \quad (60)$$

The quantity $F(r)$ can be interpreted as energy due to the energy density ϵ .

Finally, for the reason mentioned above, the presence of the coupling constant of the Gauss-Bonnet terms α produces a change in the location of these horizons. Such a change could have a significant effect in the dynamical evolution of these horizons. For nonzero α the structure of the apparent horizon is nontrivial. In general relativity noncentral singularity is always covered [35] (see also [36]). However, in the presence of the Gauss-Bonnet term we find that even the noncentral singularity is naked, in spite of being massive ($F(r > 0) > 0$).

VI. DISCUSSION

The Szekers models, depending on the sign of ϵ , are subdivided in to the quasispherical, quasiplane and quasihyperbolic [15, 16]. The geometry of the later two is not really understood [15, 16]. On the other hand the quasispherical has been rather well investigated [11–17], and it has found important applications in cosmology and gravitational collapse. The QSZ metric is a dust model which has no Killing vector [11], but contains LTB model as a spherically symmetric special case. It has been found that the LTB metric admits both naked singularities and black holes depending upon the choice of initial data. Indeed, both analytical [35–39] and numerical results [40] in dust indicate the critical behavior governing the formation of black holes or naked singularities. A similar situation also occurs in higher dimension the LTB models [31–34], and these results also carry over to QSZ spacetime [3–5]. Maeda [29] and we [28] have shown that in spherically symmetric inhomogeneous dust collapse, the effect of adding a positive α does radically alter the final fate and leads to formation of a massive timelike singularity which is prohibited in general relativity.

In this work, we have obtained an exact solution in closed form in EGB, which represents the quasispherical collapse of irrotational dust in 5D spacetime namely 5D-QSZ-EGB. The solution is nonsymmetric generalization of the spherically symmetric 5D-LTB-EGB solutions. It can be reduced to 5D-QSZ or 5D-LTB in the general relativistic case ($\alpha \rightarrow 0$). Our analysis also supports the earlier results [5], deducing that under physically reasonable initial conditions naked singularities do develop in the 5D-QSZ-EGB models, which are not spherically symmetric, and admit no Killing vectors. The second order curvature corrections changes the final fate of gravitational collapse and the nature of singularity that occurs in 5D general relativistic dust models. However, mild

departure spherical collapse can not alter the standard picture of the structure and formation singularity, since the 5D-QSZ-EGB solutions discussed here are qualitatively similar to the analogous spherical solutions. It is seen here that the Gauss-Bonnet term: (i) decelerates the collapse process (ii) alters the time of formation of singularities and the time lag between singularity formation, and (iii) modifies the apparent horizon formation and the location of apparent horizons. Our analysis to examine the nature of singularities (naked or hidden by horizon) is based on the comparison of t_{AH} (time for the formation of the apparent horizon or trapped surface) and t_c (time for the formation of a central singularity). In QSZ the singularity can be directionally naked [5]. Hence, it would be necessary to investigate in further details the final fate of the inhomogeneous dust collapse in the EGB theory in order to bring out explicitly the difference in global nakedness when we depart from spherically symmetric [42].

The conjecture that such a singularity from a regular initial surface must always be hidden behind an event horizon, called cosmic censorship conjecture (CCC) was proposed by Penrose [41]. The CCC forbids the existence of naked singularities. Despite almost 30 years of effort, we are far from a general proof of CCC (for recent reviews and references, see [43]). But, significant progress has been made in trying to find counter examples to CCC. Our analysis, as in the 5D-LTB-EGB [28], shows that there exists a regular initial data which leads to a naked singularity and hence in our nonspherical case also the CCC is violated. The usefulness of these models is that they do offer an opportunity to explore the properties of singular spacetime. The investigation of a mild departure from standard spherical symmetry models may be valuable in attempts to put CCC in concrete mathematical form. Finally, studying such models which lacks symmetry is important, so that one can check which properties of gravitational collapse are preserved. This may also helps us to bring out some universal features in the theory of gravitational collapse [42]. We have shown here that there exist regular initial data which leads to a naked singularity violating CCC. However, this may not be a serious threat to CCC because of the following two reasons viz. (i) The matter considered here is dust which is only an effective, macroscopic approximation to a fundamental description of matter [44]. (ii) In general relativity the energy-momentum tensor given by Eq. (8) satisfies the weak energy condition. However, this may not be true in EGB because the Gauss-Bonnet term itself violates the energy condition.

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